



Suspension choices for the Einstein Telescope

Giles Hammond

Institute for Gravitational Research

University of Glasgow

giles.hammond@glasgow.ac.uk



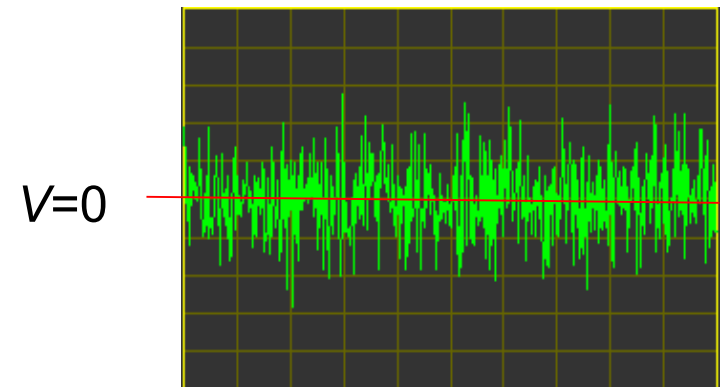
The Einstein telescope project, 31st January 2018, Liege University

- Suspension thermal noise
 - Brownian noise
 - Thermoelastic noise
- 2nd generation suspensions in fused silica
- ET warm suspensions
- ET cold suspensions
- Summary

- Thermal noise is the statistical movement of particles driven by thermal energy $k_B T$
- The fluctuation of the surface can be produced by two possible mechanisms:
 - Brownian motion of the surface – **Brownian thermal noise**
 - statistical temperature fluctuations within the test mass cause local changes of the surface position (thermal expansion coefficient/Young's modulus) – **Thermoelastic noise**
- Brownian noise is analogous to Johnson voltage noise in resistors, the mean squared fluctuation is non-zero

$$\langle V \rangle = 0$$

$$\langle V^2 \rangle = 4k_B T R \Delta f$$



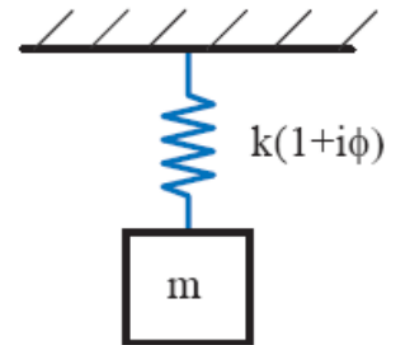
- The Fluctuation Dissipation Theorem provides the link between the spectral density of a fluctuating displacement S_x and the mechanical impedance $Z(\omega)$ of the system:

$$S_x(\omega) = \frac{4k_B T}{\omega^2} \Re \left[\frac{1}{Z(\omega)} \right] \quad m^2 / Hz$$

thermal displacement noise mechanical impedance (F / \dot{x})

- For a harmonic oscillator we find

$$S_x(\omega) = \frac{4k_B T}{\omega m} \left[\frac{\phi(\omega) \omega_0^2}{(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi(\omega)^2} \right]$$



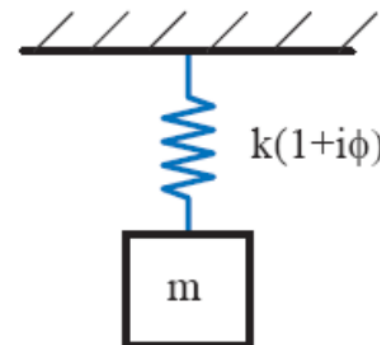
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thermal displacement noise mechanical impedance (F / \dot{x})

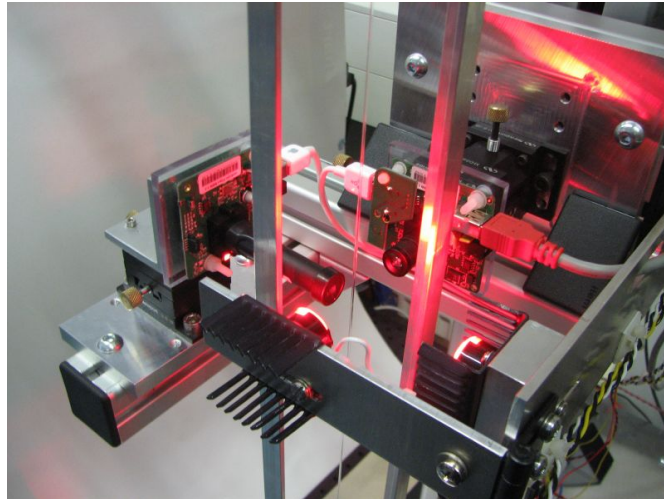
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$$S_x(\omega) = \frac{4k_B T}{\omega m} \left[\frac{\phi(\omega) \omega_0^2}{(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi(\omega)^2} \right]$$

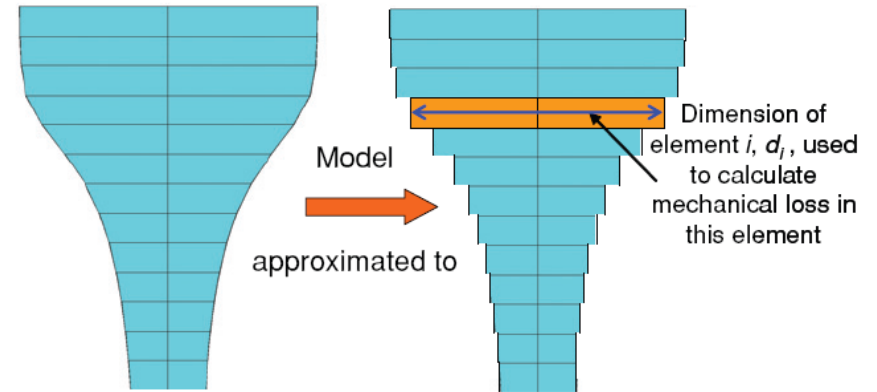


- ET HF: Low loss
- ET LF: Low temperature and/or low loss

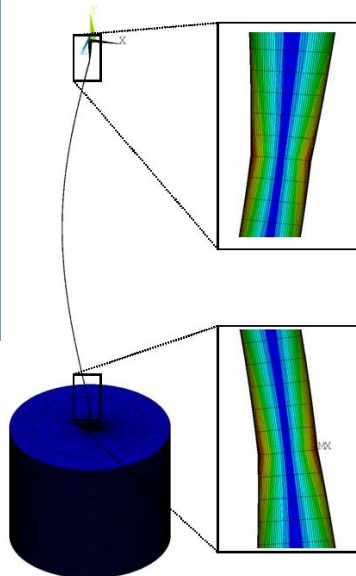
Suspension Thermal Noise



- Profile fused silica fibre

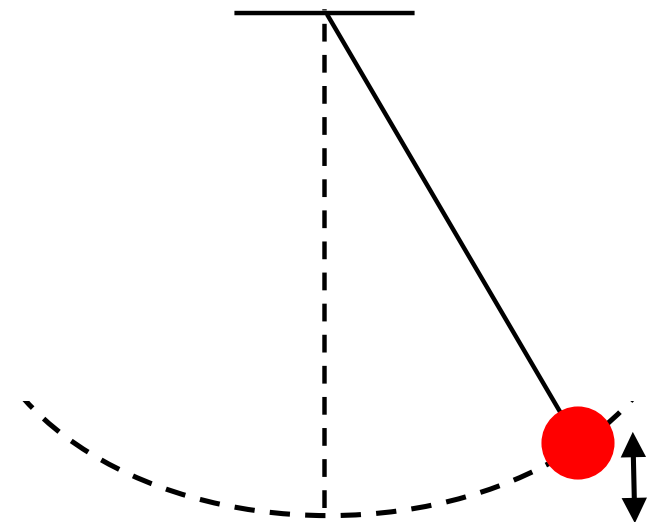


- Extract elastic strain energy and modal frequencies from ANSYS



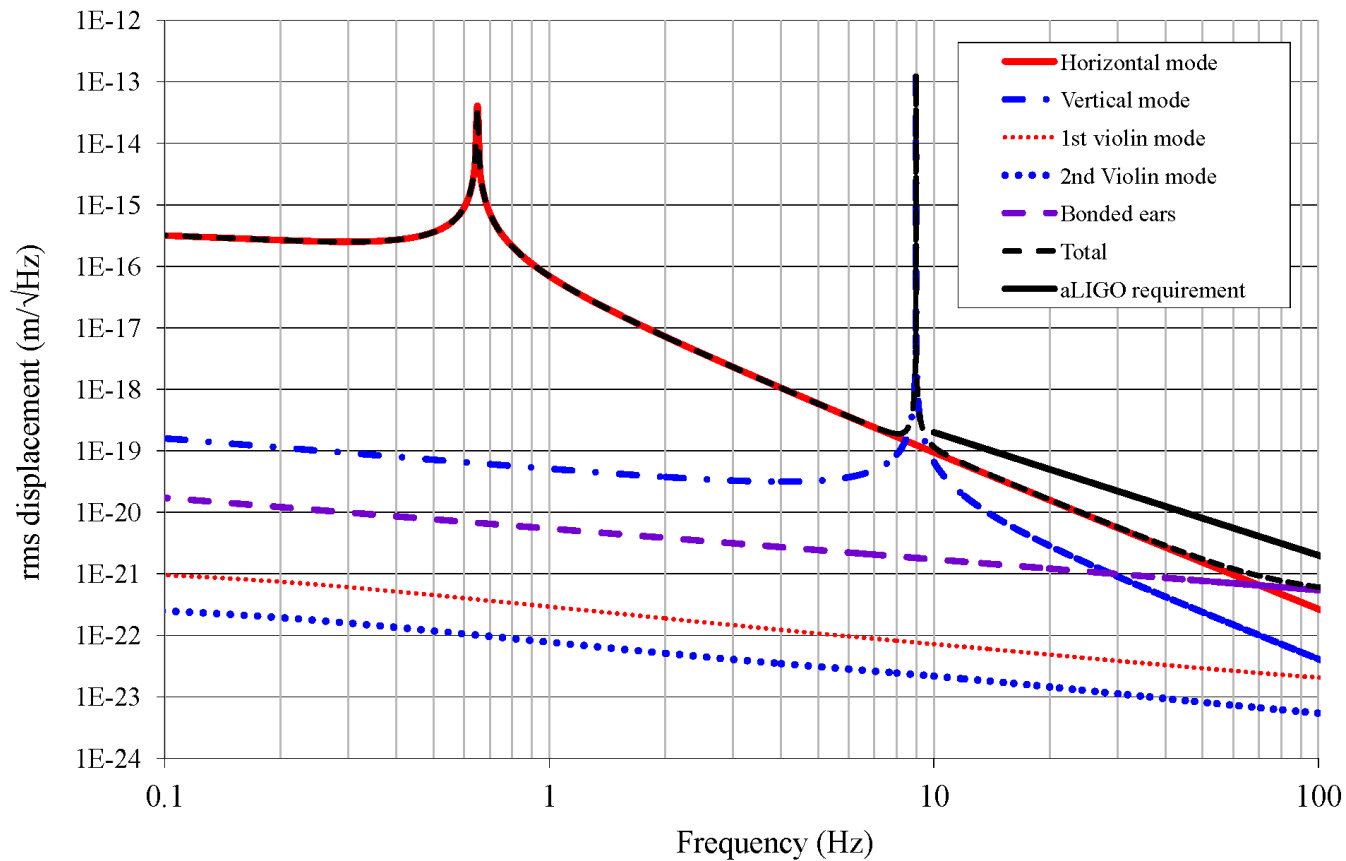
- Loaded suspension fibres the majority of energy in gravity \Rightarrow dilute loss due to elastic energy

$$D = \frac{E_{\text{total}}}{E_{\text{elastic}}} \approx \frac{k_{\text{gravity}}}{k_{\text{fibre}}} \approx 2L \sqrt{\frac{T}{YI}}$$



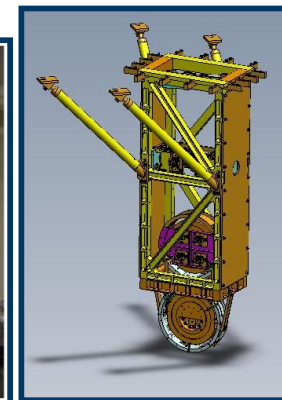
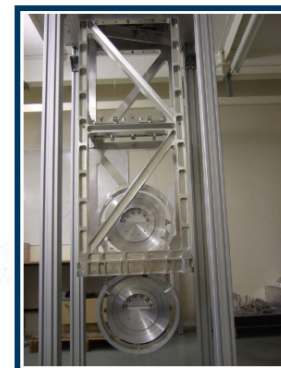
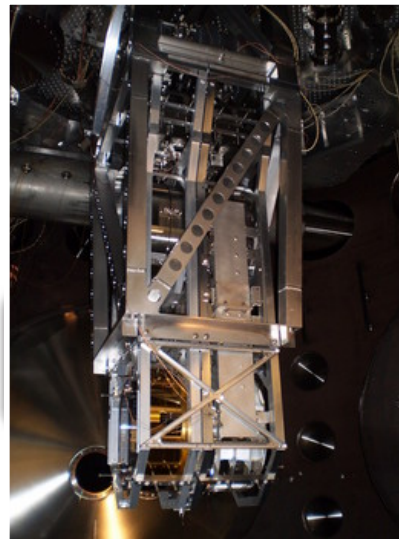
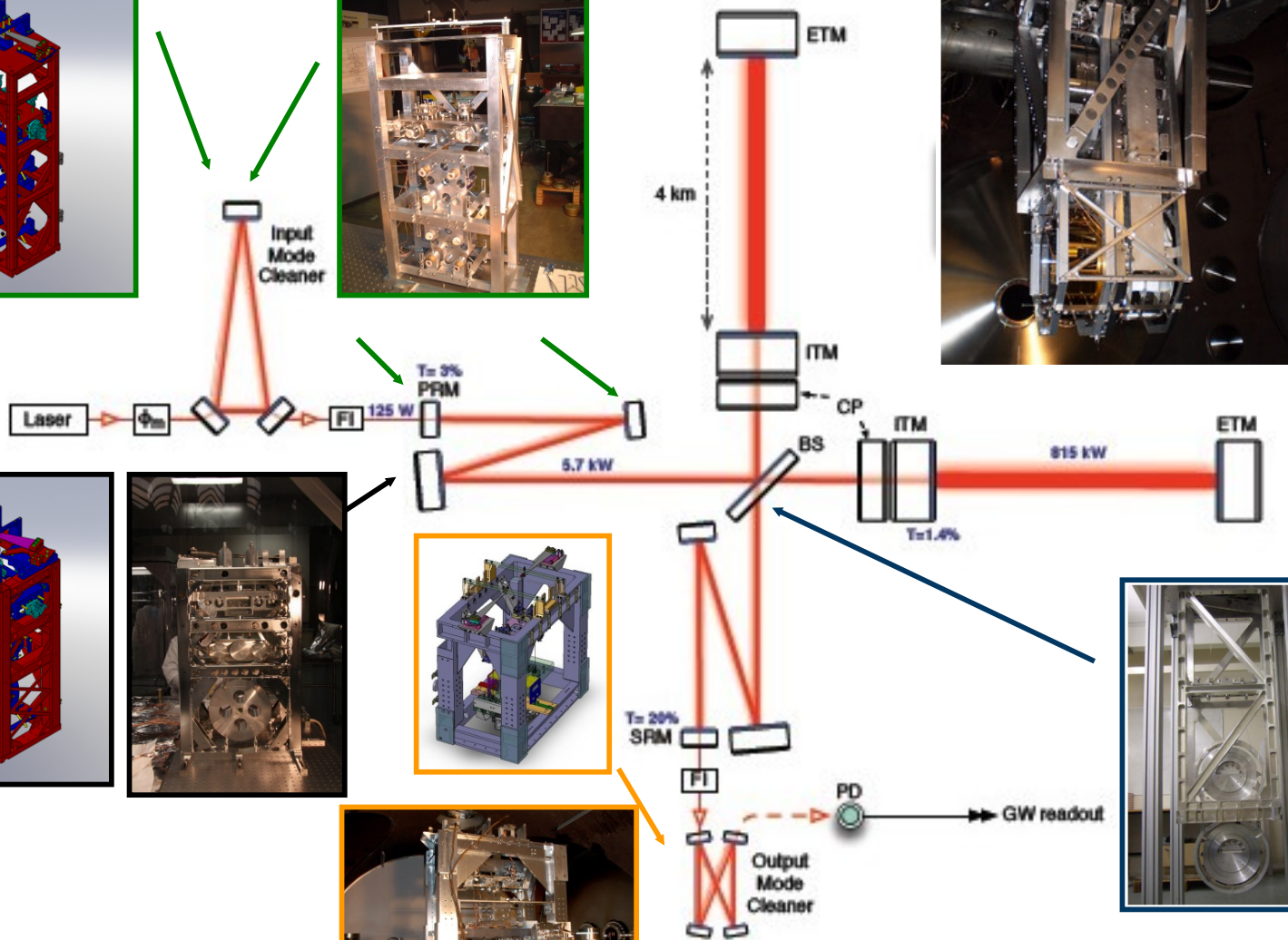
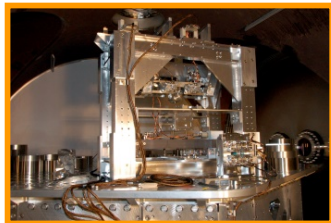
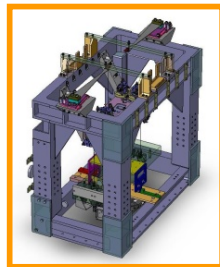
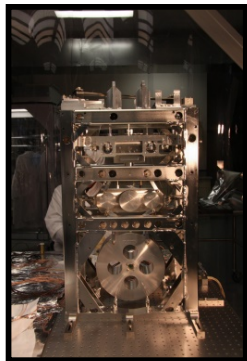
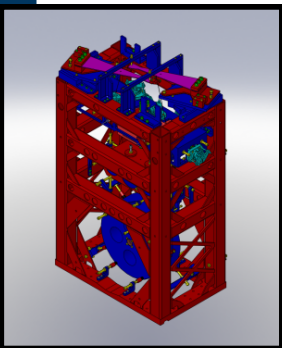
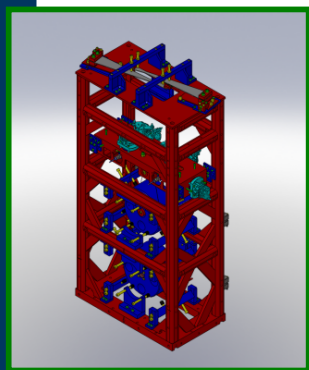
$$S_x(\omega) = \frac{4k_B T}{\omega m} \left[\frac{\phi(\omega)\omega_0^2}{(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi(\omega)^2} \right]$$

$$D \approx 2L \sqrt{\frac{T}{YI}}$$

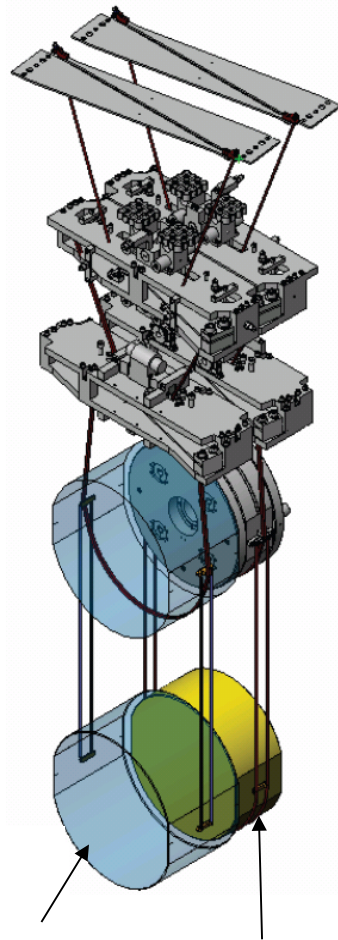


Fused silica + dilution of 90 provide thermal noise performance of 10^{-19} m/√Hz at 10Hz

Suspensions in aLIGO

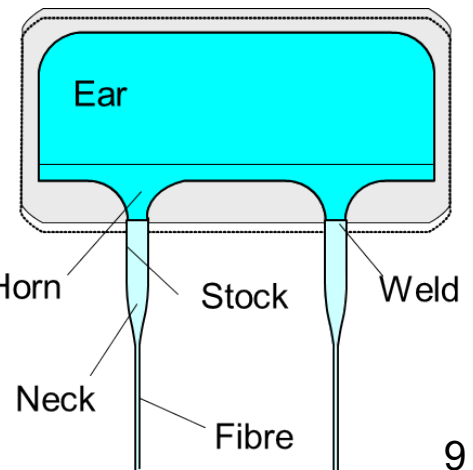
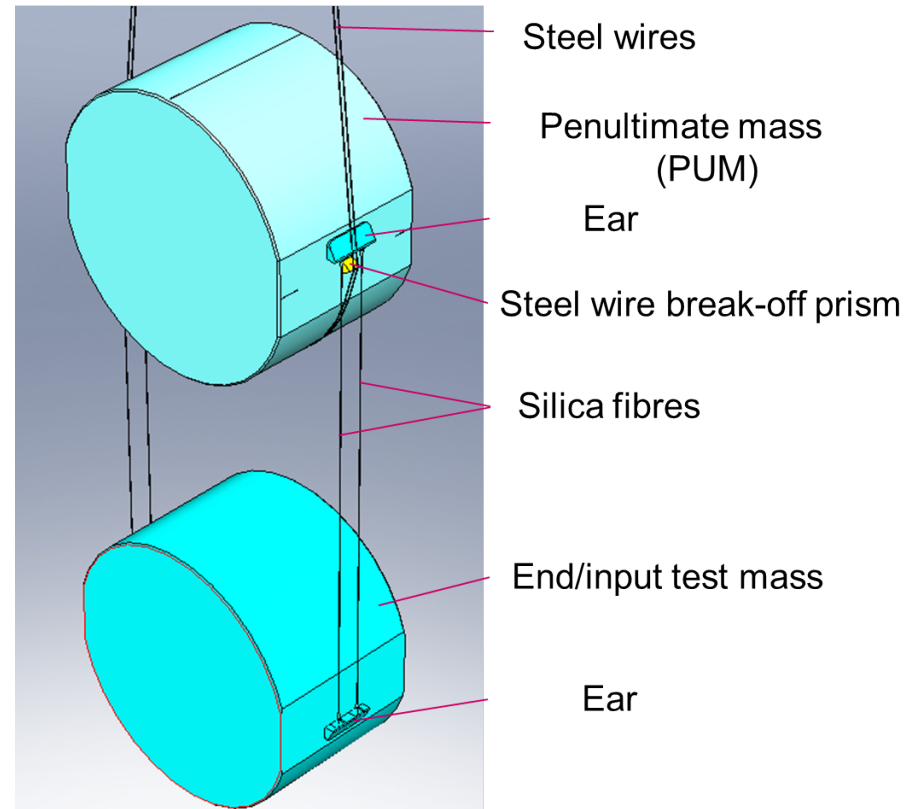


aLIGO Quadruple Suspension



40kg silica test mass

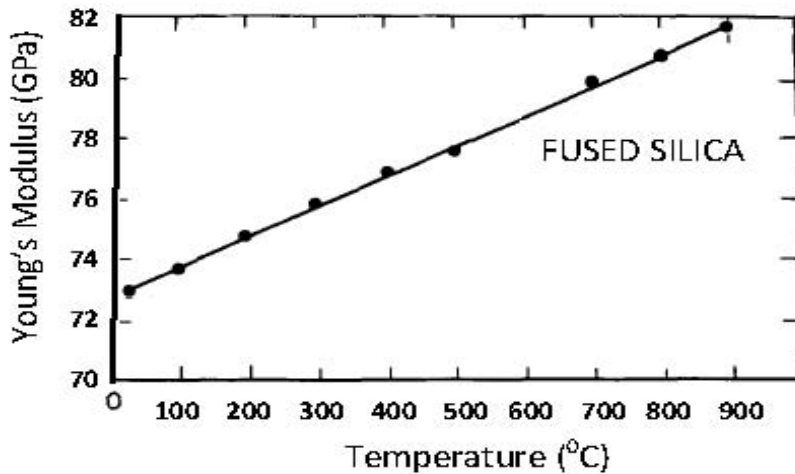
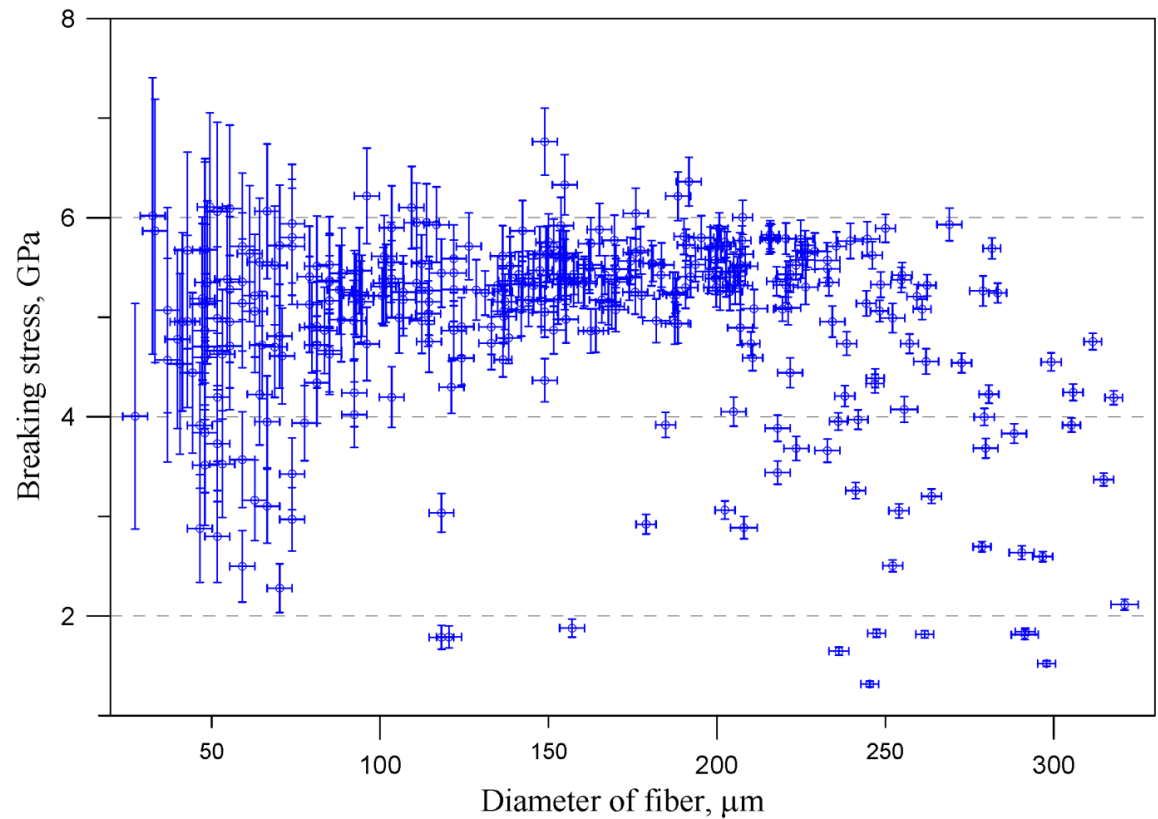
parallel reaction chain for control



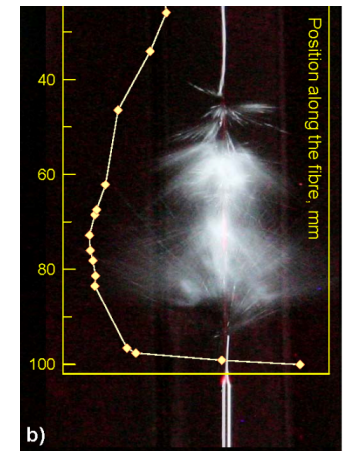
- Thermal expansion coefficient

$$\alpha_{eff} = \left(\alpha - \frac{\sigma}{Y^2} \frac{dY}{dT} \right)$$

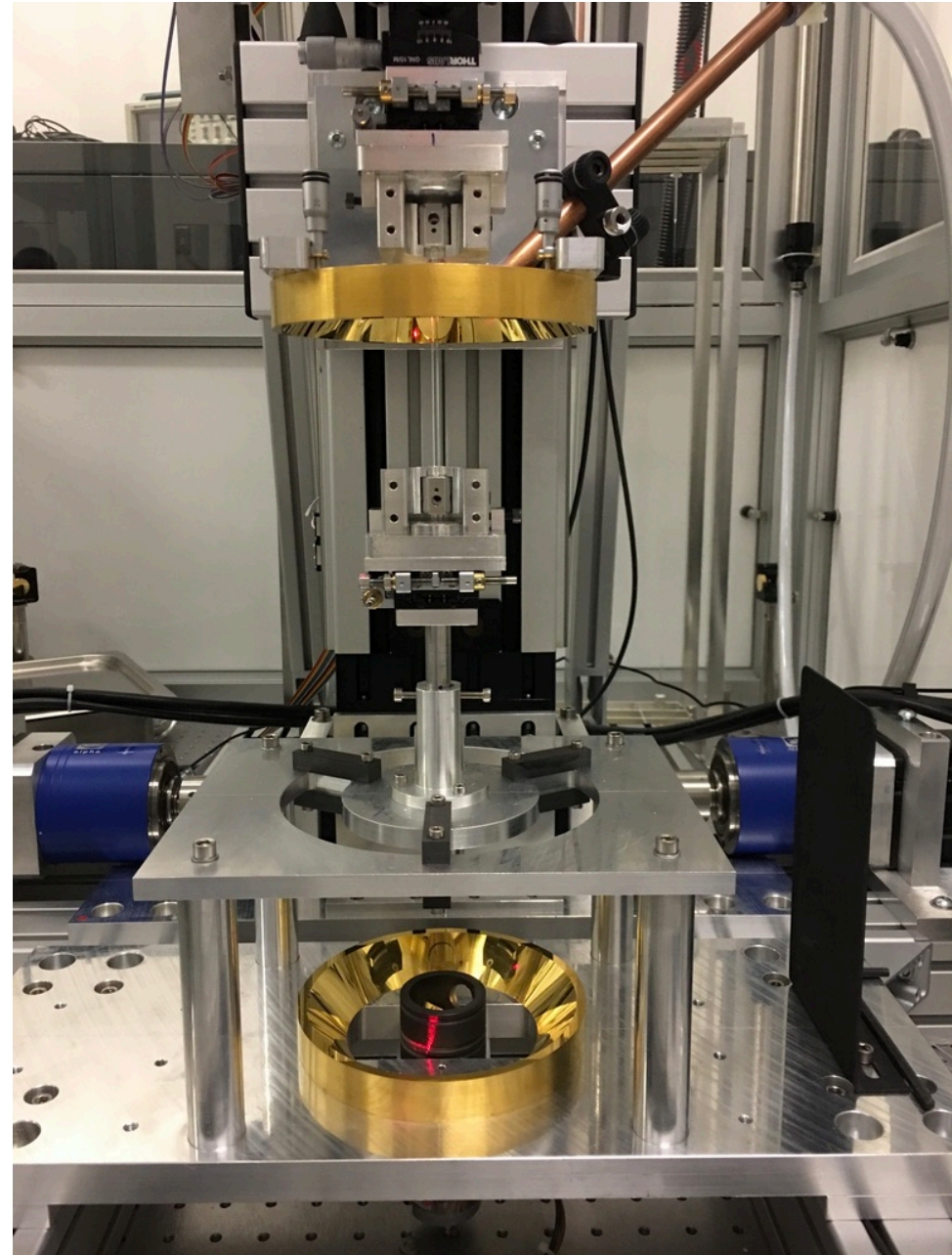
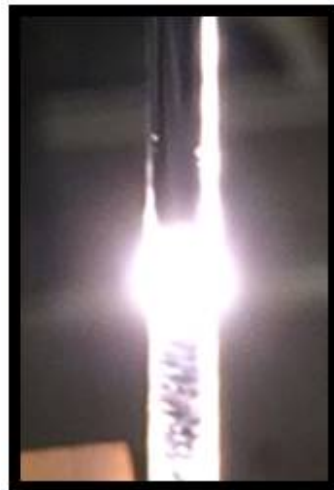
- Used to null thermoelastic noise around 10 Hz



- Ultra low mechanical loss
- Can pull into fibres/weld
- Strong (tensile stress 4-5GPa)
- “zero” thermal expansion

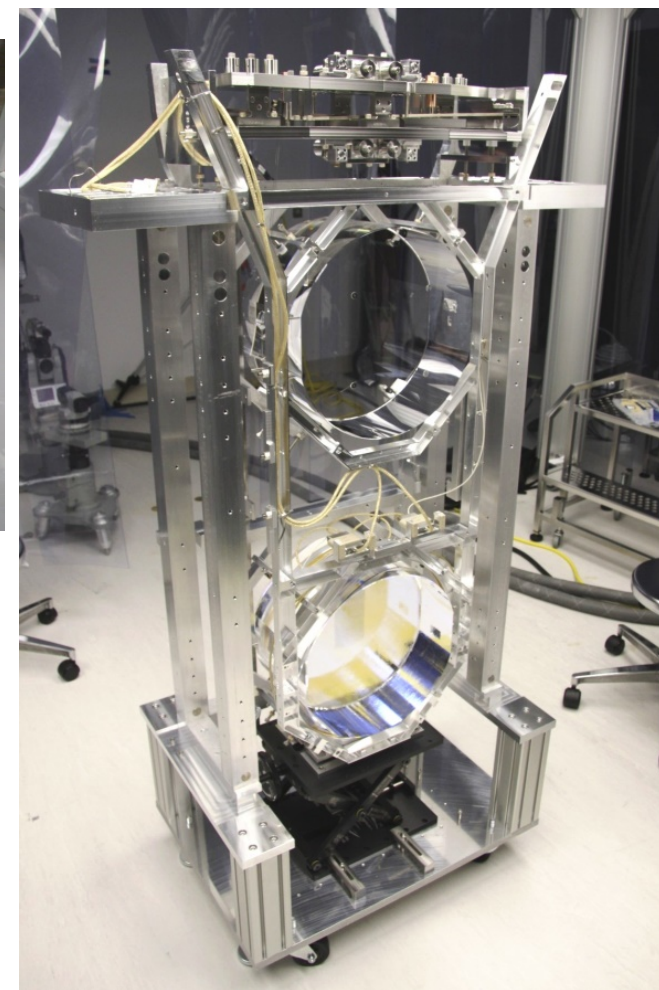
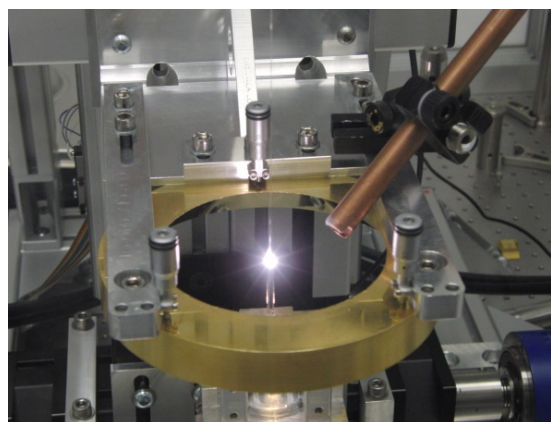
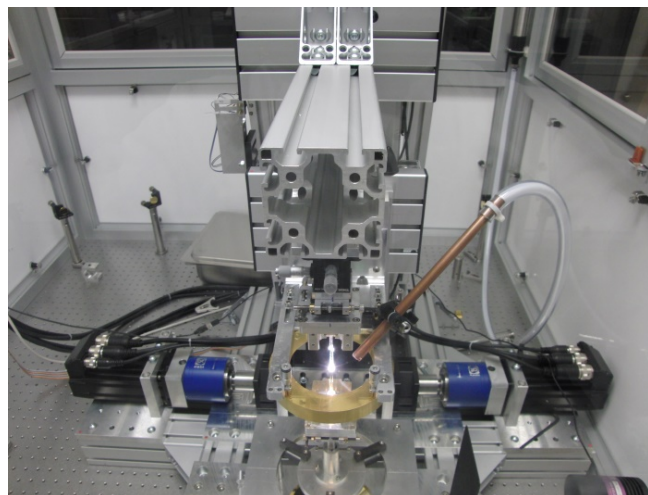
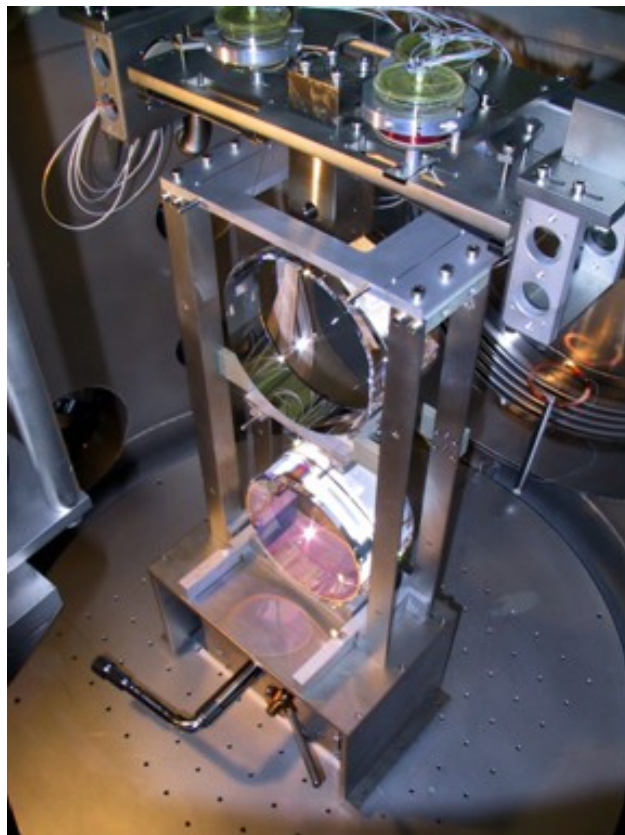


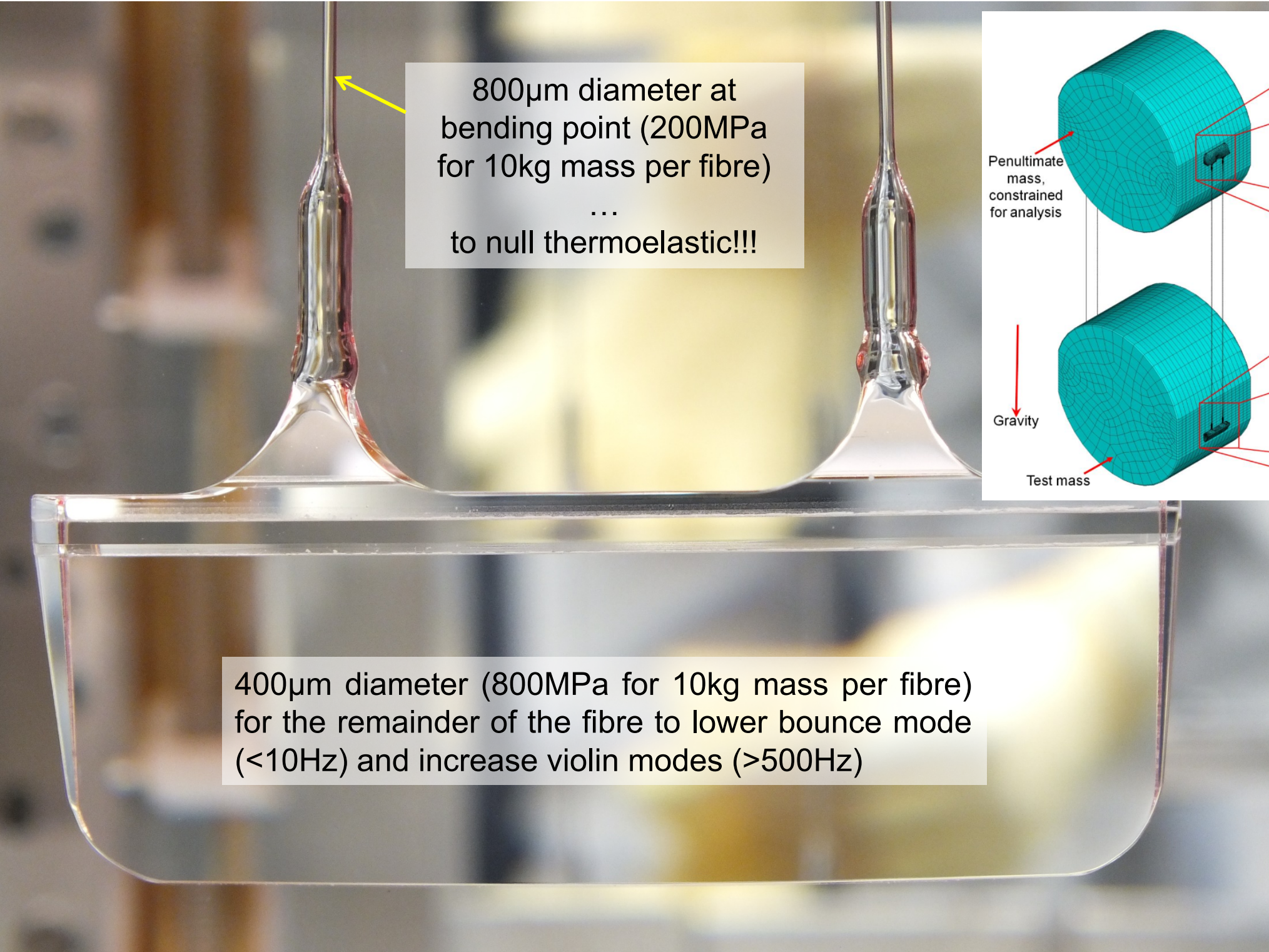
- 100W CO₂ laser to heat fused silica
- Rotating mirror directs CO₂ beam out to the conical mirror to then create a cylindrical beam to the feed conical mirror.
- Pulling stage moves up to produce fibre.



Monolithic Suspensions

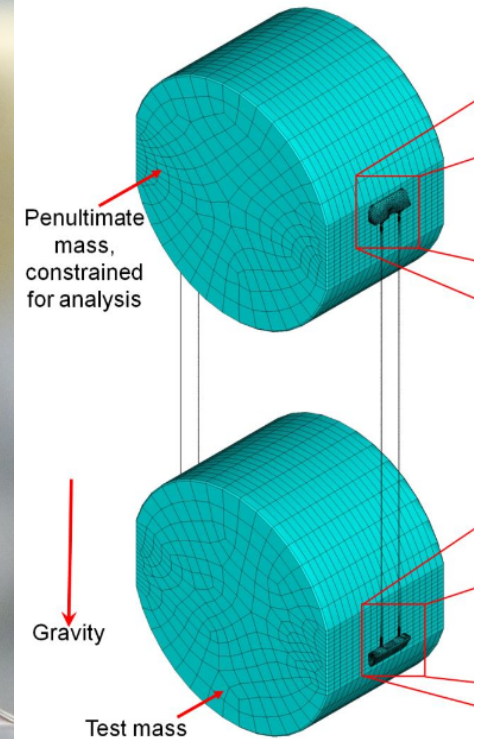
- Monolithic suspensions & signal recycling pioneered in GEO-600 → upscaled to aLIGO





800 μ m diameter at
bending point (200MPa
for 10kg mass per fibre)

...
to null thermoelastic!!!



400 μ m diameter (800MPa for 10kg mass per fibre)
for the remainder of the fibre to lower bounce mode
(<10 Hz) and increase violin modes (>500 Hz)

ET Warm Suspensions

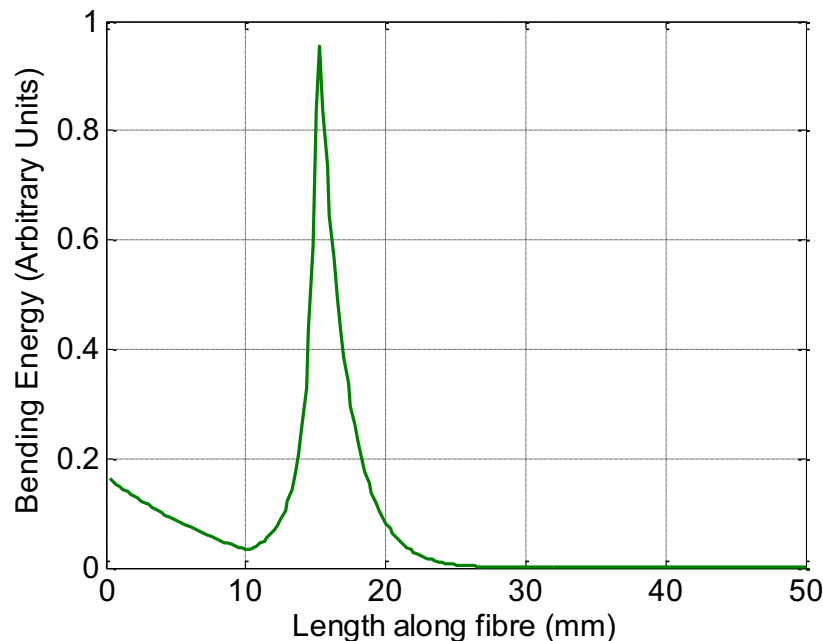
ET Warm Suspensions

- A variety of techniques will need to be used to further improve room temperature suspension thermal noise, needed for ET-HF

$$\phi_{total}(\omega) = \frac{1}{D} \left[\frac{E_1}{E_{elastic}} \phi_1(\omega) + \frac{E_2}{E_{elastic}} \phi_2(\omega) + \dots + \frac{E_n}{E_{elastic}} \phi_n(\omega) \right]$$

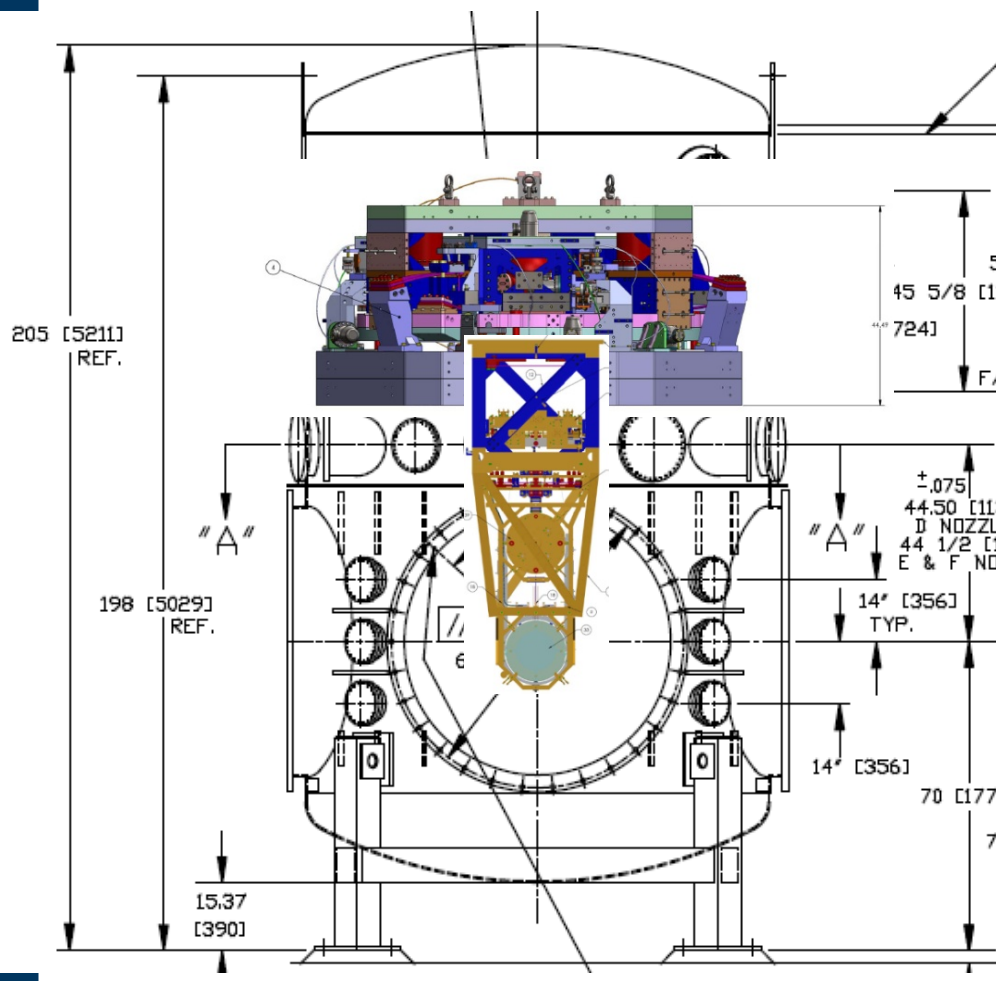


$$S_x(\omega) = \frac{4k_B T}{m\omega} \left(\frac{\omega_o^2 \phi_{total}(\omega)}{\omega_o^4 \phi_{total}^2(\omega) + (\omega_o^2 - \omega^2)^2} \right)$$

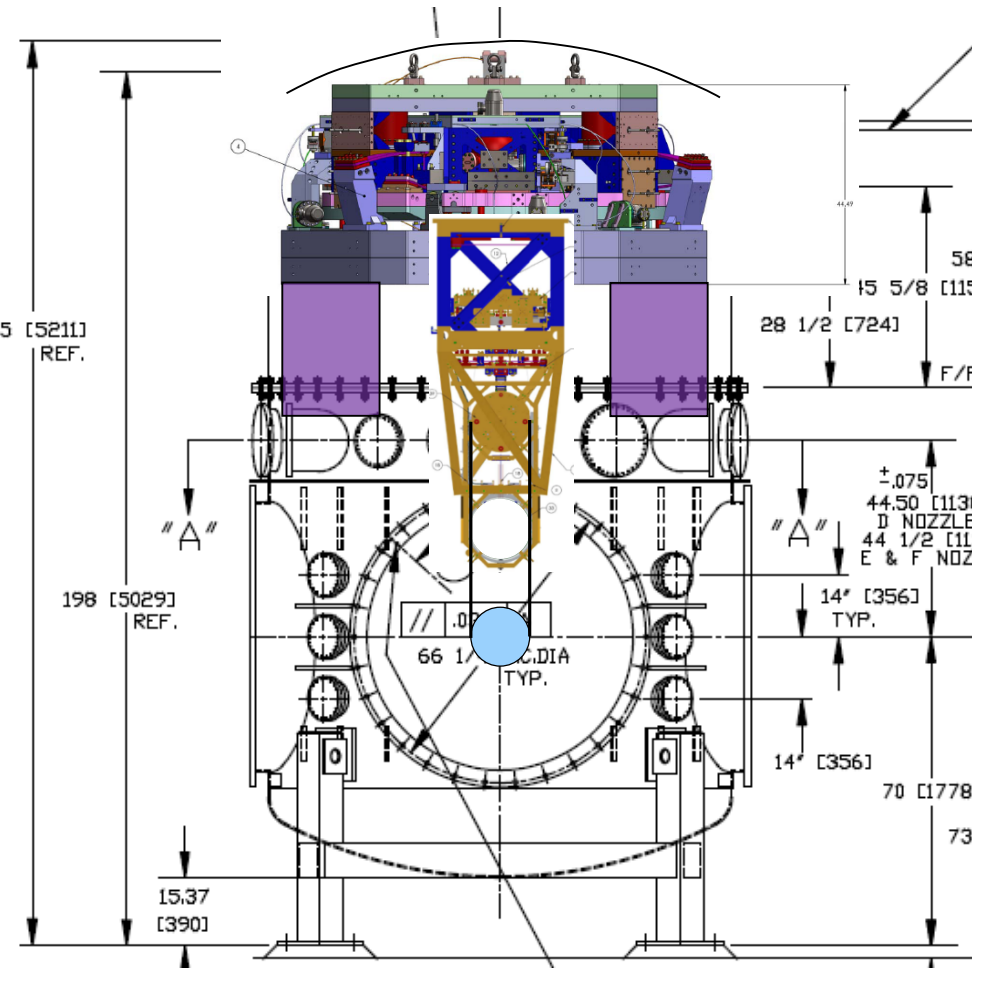


- Reduce surface loss and weld loss in suspension
- Longer suspensions to improve dilution
- Shorter fibre neck => reduce energy distribution up the neck
- Shorter stock length => reduce energy distribution up the neck
- Pull from thicker stock (3mm→5mm)

ET Warm Suspensions



aLIGO (0.6 m final stage)

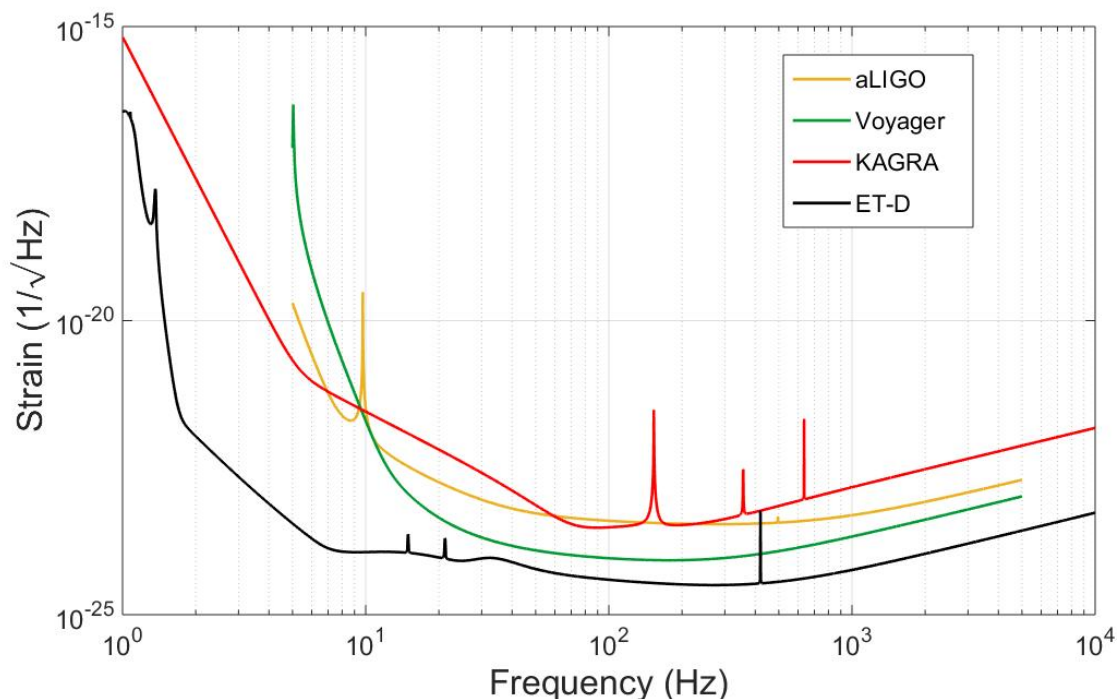
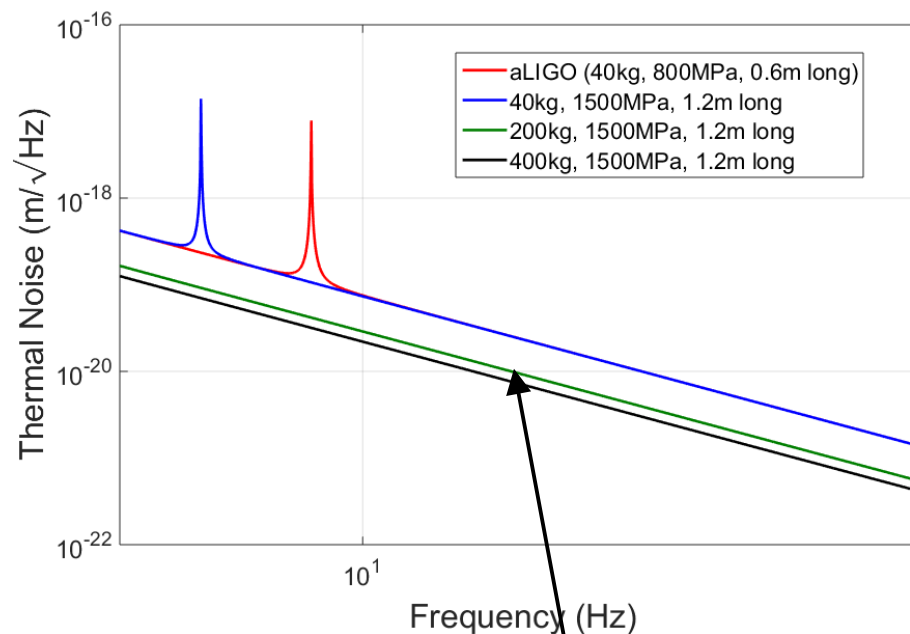


LIGO-3 (≈1.2 m final stage)

$$D \approx 2L \sqrt{\frac{T}{YI}}$$

Sensitivity Improvement

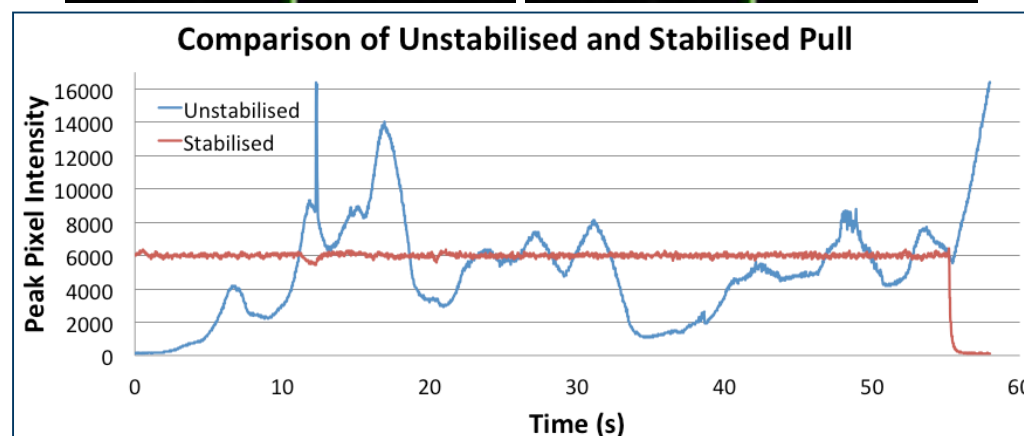
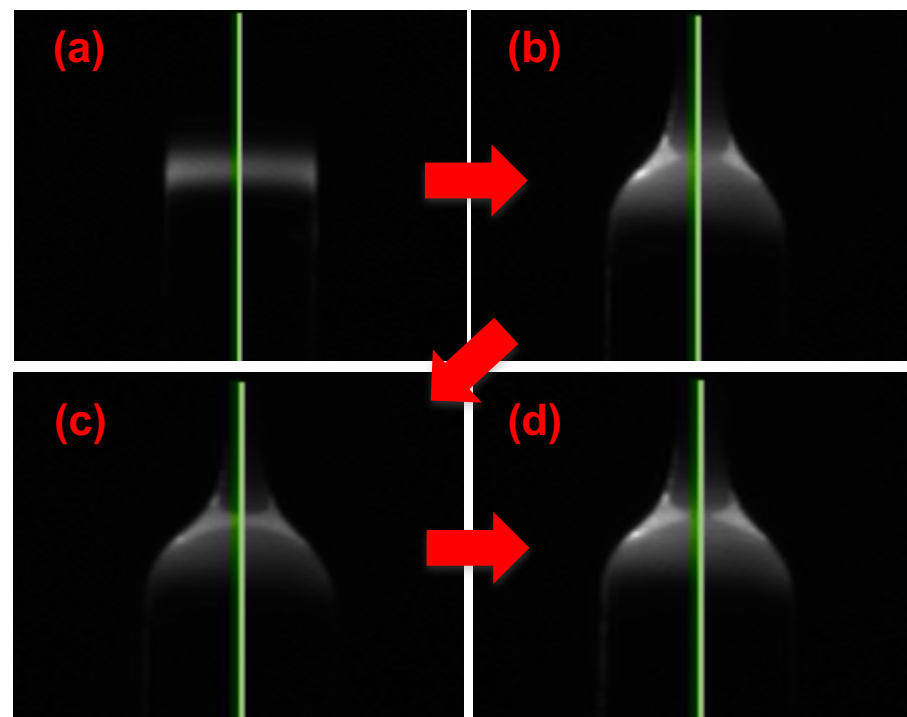
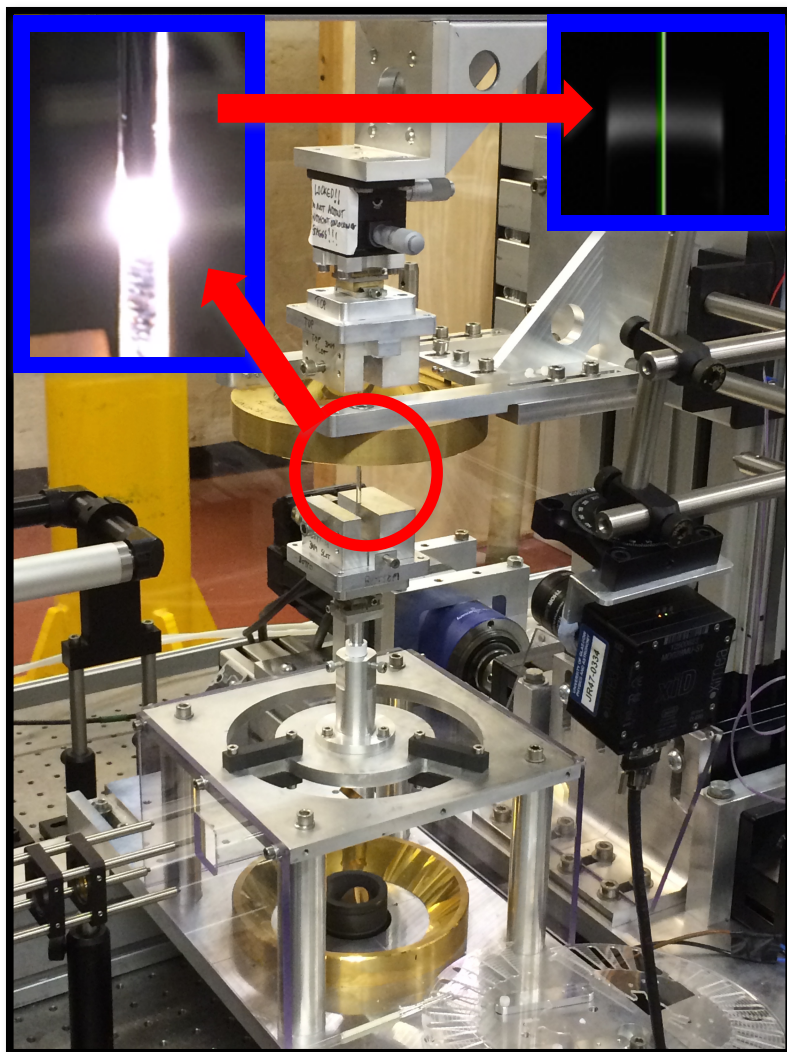
- Heavier test mass (200 kg)
- Longer suspensions (1.2m-2m) to improve dilution
- Higher fibre stress to manage bounce/violin mode



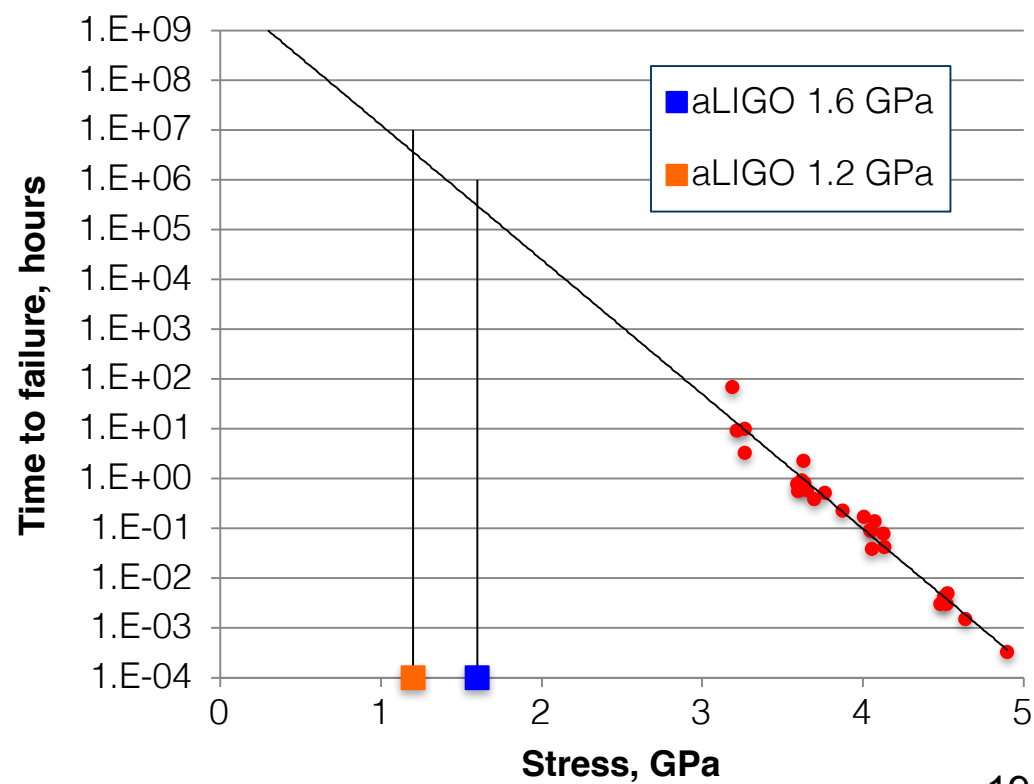
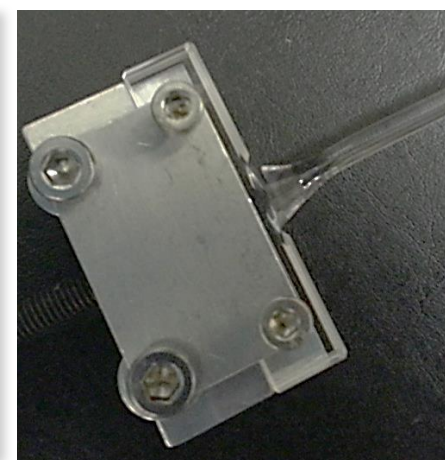
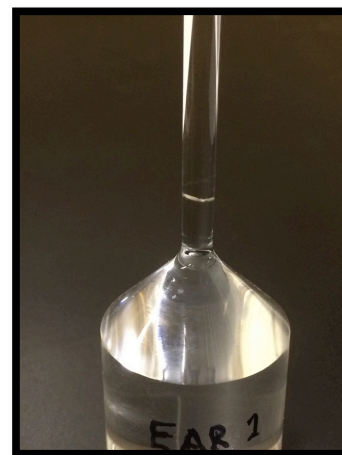
For 10km arms and 1.2m silica stage, strain noise of $9 \times 10^{-25} 1/\sqrt{Hz}$ at 30Hz (consistent with high frequency ET-D)

Laser Stabilisation

- Require heavy test masses operating at higher fibre stress => improve robustness



Heavy Test Mass Suspension



ET Cold Suspensions

ET Cold Suspensions

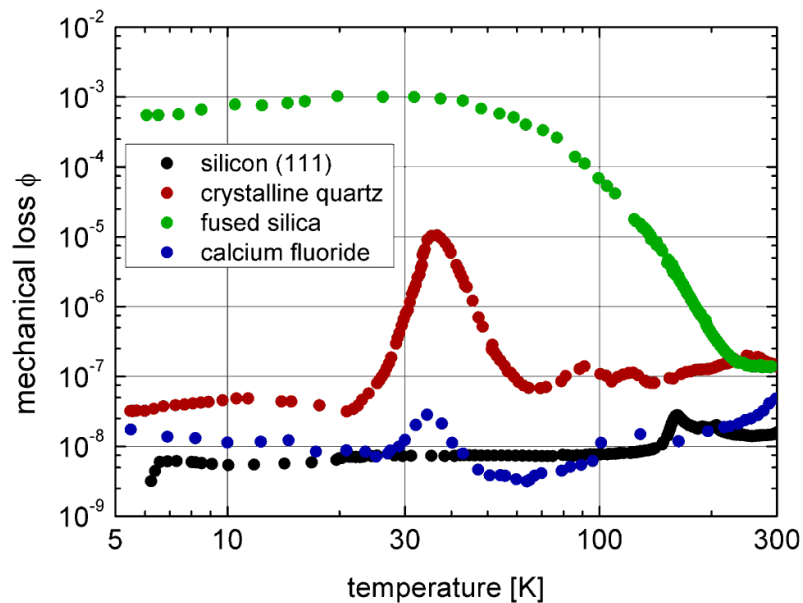
- Crystalline materials exhibit high thermal conductivity and low mechanical loss
- Trade-off in circulated power and heat extraction techniques:
 - > 100K radiative cooling
 - < 20K conduction cooling)

Sapphire @ 20K

Silicon @ 123K** or 20K

** single detector @123K

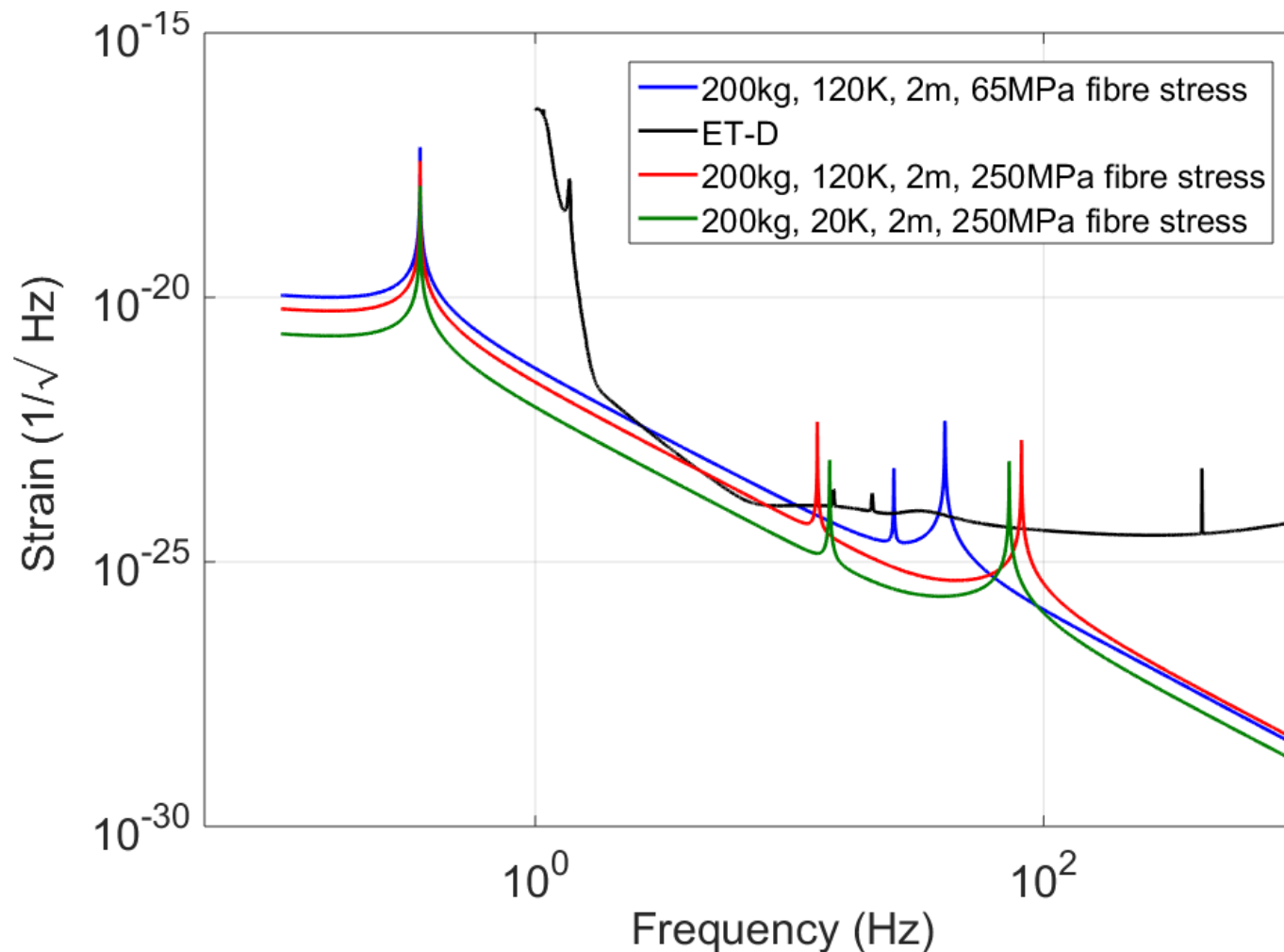
dual detector at 30K/20K



Property	Sapphire	Silicon
mechanical loss		
mechanical strength	Good, for pristine material	Good, for pristine material
optical material	1064nm, 40ppm/cm	$\geq 1550\text{nm}^*$, <5 ppm/cm for mCz**
thermal conductivity	$\approx 3\text{kW/mK}$ @ 20K	$\approx 5\text{kW/mK}$ @ 20K
Polishing	Hard material	
size availability	23kg possible	semiconductor industry/purity

Cold Upgrades: Suspensions

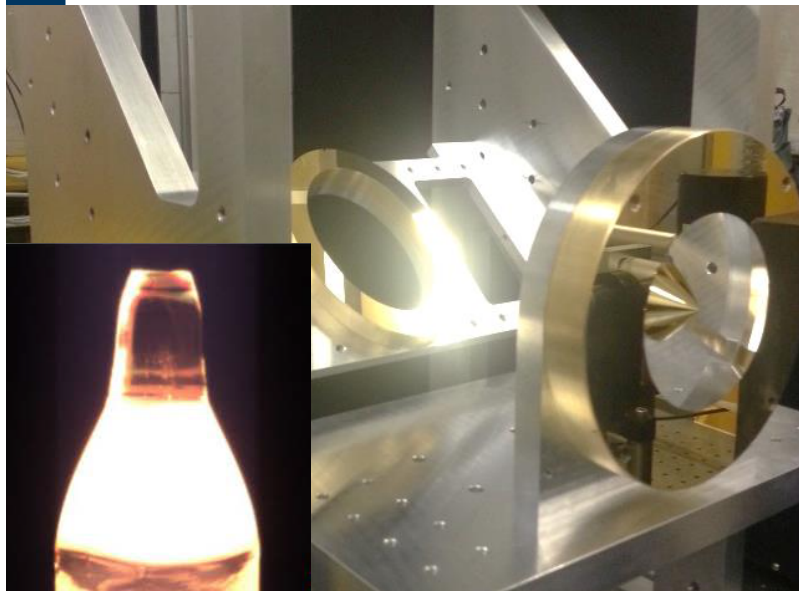
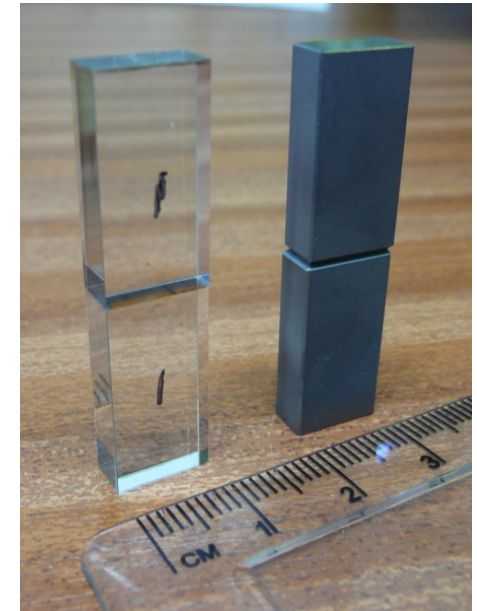
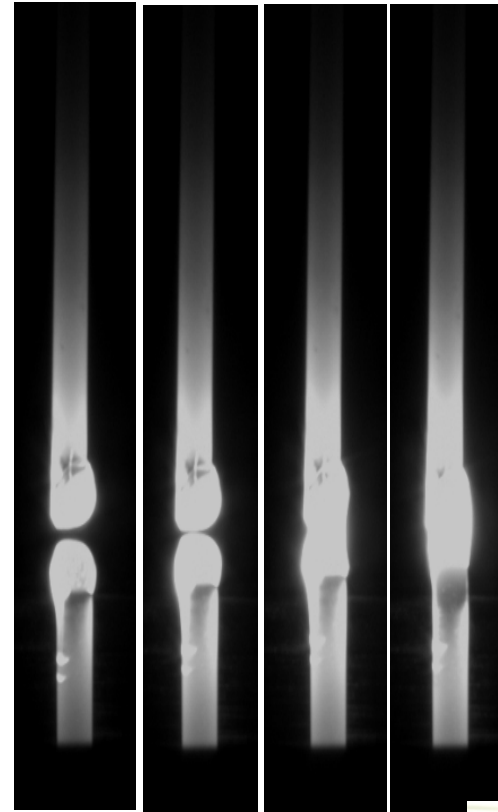
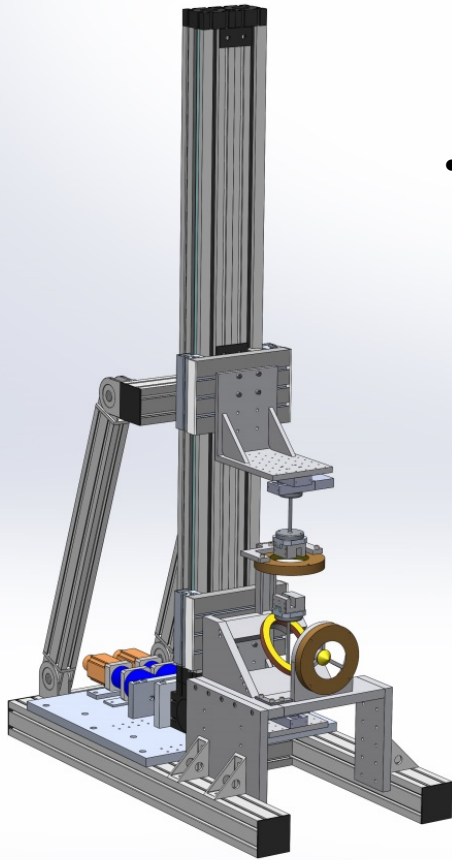
- Tensile strength tests of Silicon suggest values of 200MPa-300MPa for a variety of samples which have been mechanically polished, etched or oxidized (fused silica is 4GPa-5GPa) => need to grow pristine material?



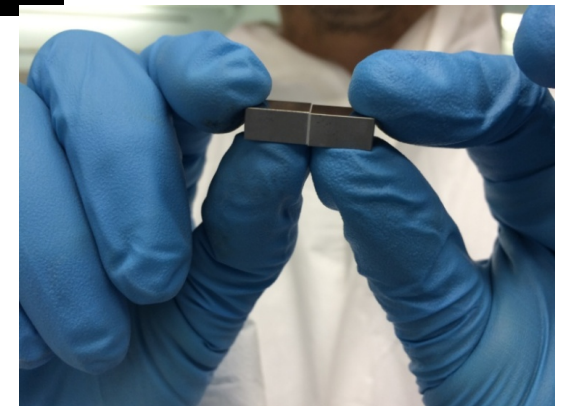
Crystal Growth & Bonding

- Silicon suspensions
- Welding sapphire

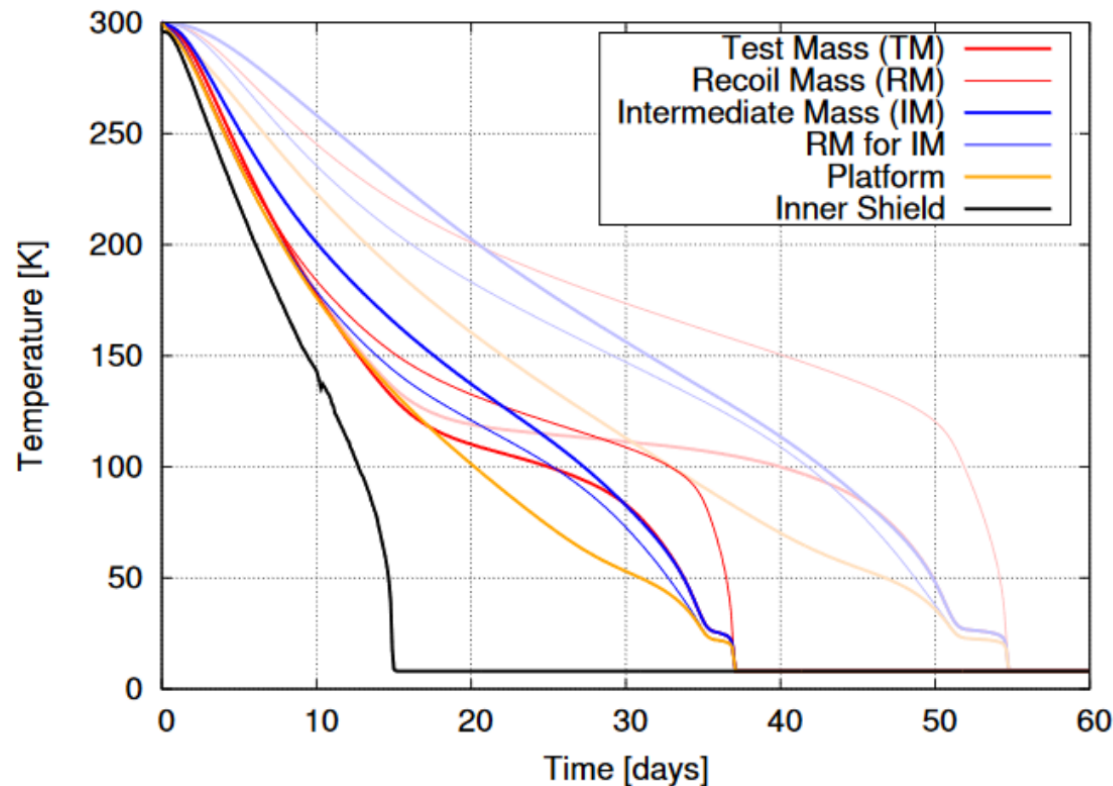
Cumming et al, Class.
Quant. Grav., 31,
025017, 2014



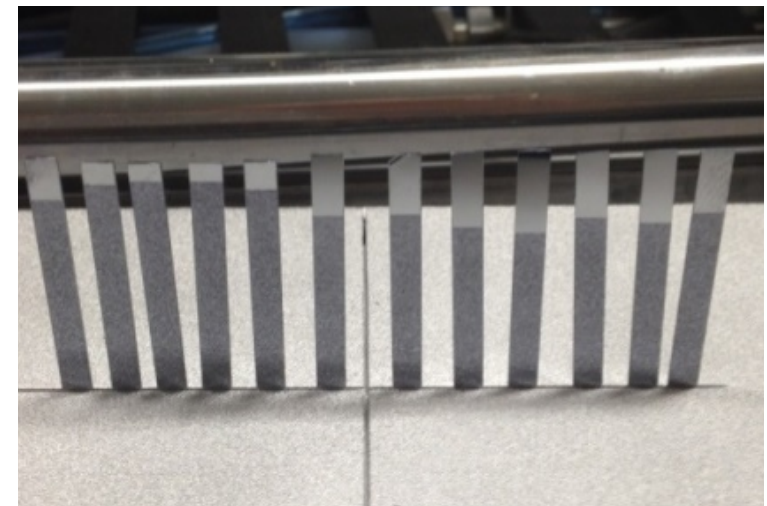
- Hydroxide-catalysis bonding of sapphire/silicon: quasi-monolithic suspensions



- DLC has potential applications for;
 - cryogenic applications (baffle tubes/UHV compatible)
 - protective coatings on springs/fibres in the suspension



Cooling time for KAGRA (solid: with
DLC, greyed: without DLC)

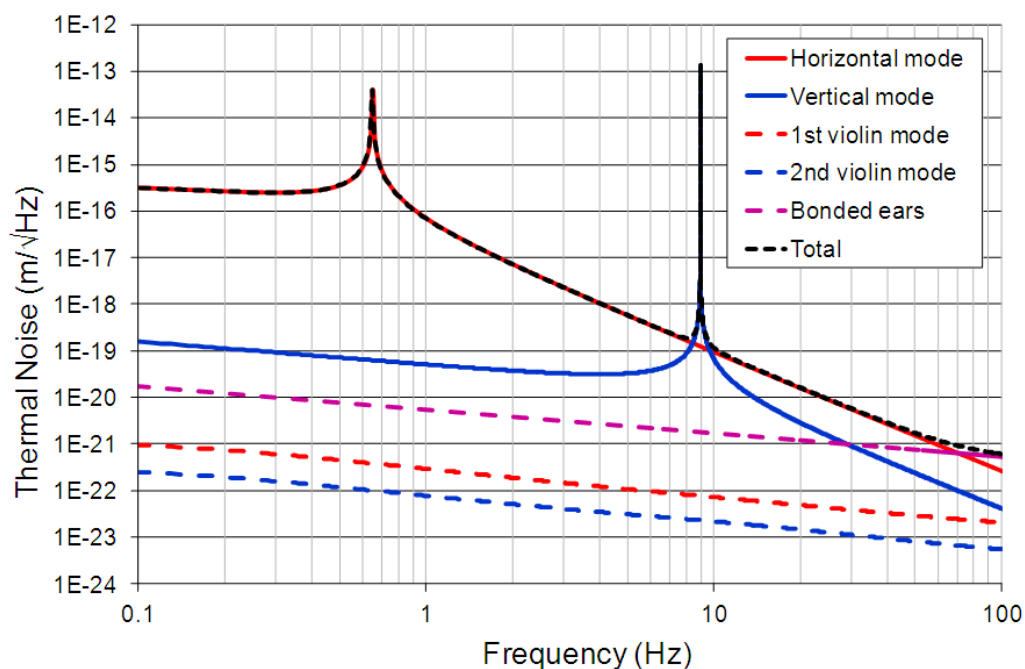


Vertical mounting of silicon
flexures for DLC coating tests

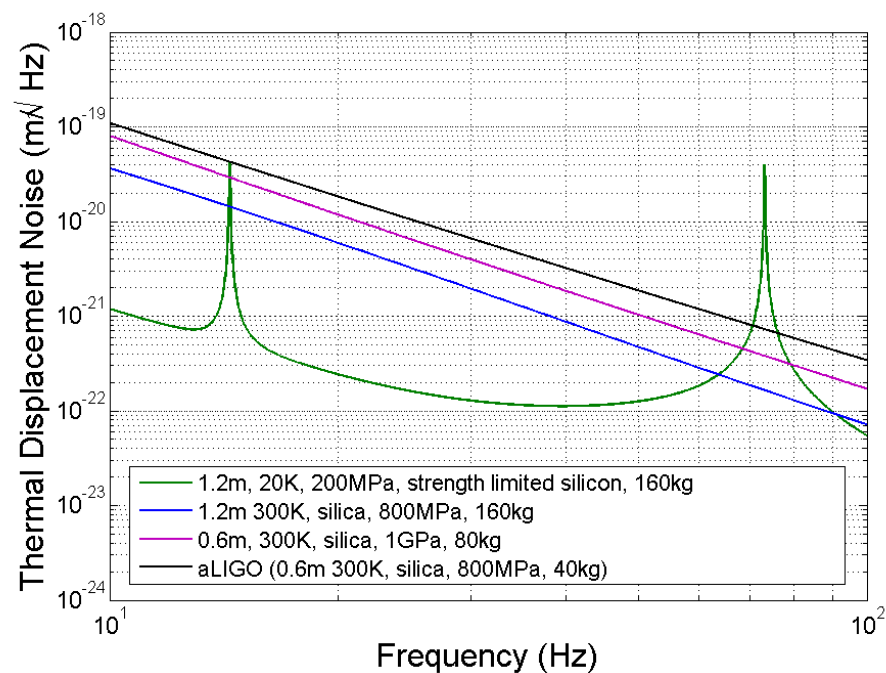
(see DCC G1700069 for more info)

Summary

- Warm upgrades with fused silica offer a well developed technology and improvements in strain sensitivity. Thermal noise improvement of x3.
- Cryogenic upgrades offer potential improvements in thermal noise of >20. Silicon at 120K is quite interesting as it has zero thermoelastic loss.
- Need R&D to develop warm/cold prototypes, and develop the necessary fabrication techniques

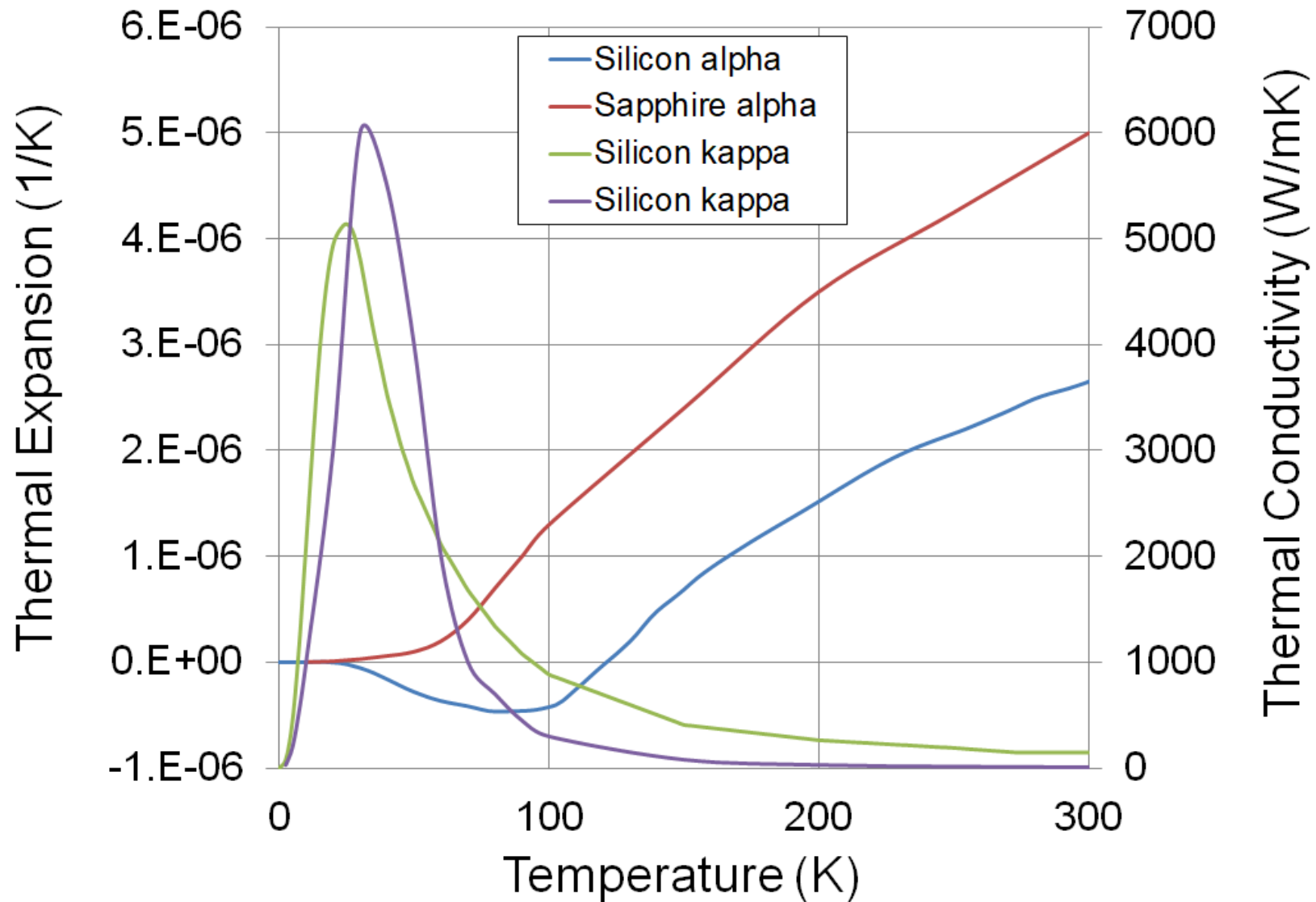


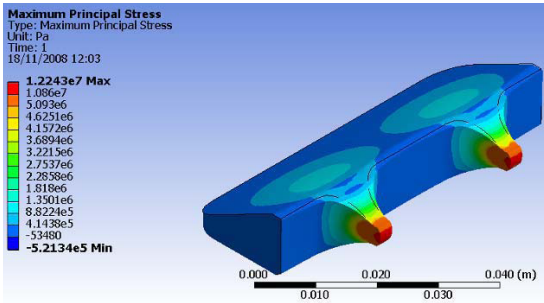
aLIGO: $10^{-19}\text{m}/\sqrt{\text{Hz}}$ at 10Hz



Extra Slides

- Properties of Sapphire/Silicon





Suspension Thermal Noise

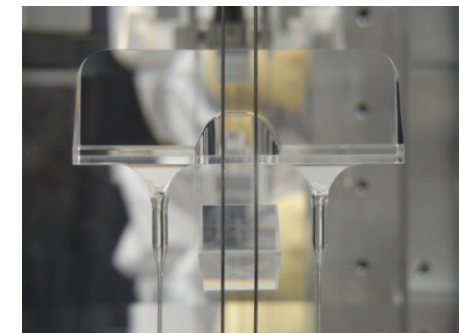


- Use the following loss terms to model the welds, ear horns and fibres

$\phi_{\text{bulk}} = 1.2 \times 10^{-11} f^{0.77}$	$\phi_{\text{TE}}(\omega) = \frac{YT}{\rho C} \left(\alpha - \sigma_o \frac{\beta}{Y} \right)^2 \left(\frac{\omega\tau}{1 + (\omega\tau)^2} \right)$
$\phi_{\text{surface}} \approx \frac{8h\phi_s}{d}$	$\phi_{\text{weld}}(\omega) = 5.8 \times 10^{-7}$

$$\phi_i(\omega) = (\phi_{\text{bulk},i}(\omega) + \phi_{\text{TE},i}(\omega) + \phi_{\text{surface},i}(\omega) + \phi_{\text{weld},i}(\omega))$$

$$\phi_{\text{total}}(\omega) = \frac{1}{D} \left[\frac{E_1}{E_{\text{elastic}}} \phi_1(\omega) + \frac{E_2}{E_{\text{elastic}}} \phi_2(\omega) + \dots + \frac{E_n}{E_{\text{elastic}}} \phi_n(\omega) \right]$$



$$S_x(\omega) = \frac{4k_B T}{m\omega} \left(\frac{\omega_o^2 \phi_{\text{total}}(\omega)}{\omega_o^4 \phi_{\text{total}}^2(\omega) + (\omega_o^2 - \omega^2)^2} \right)$$

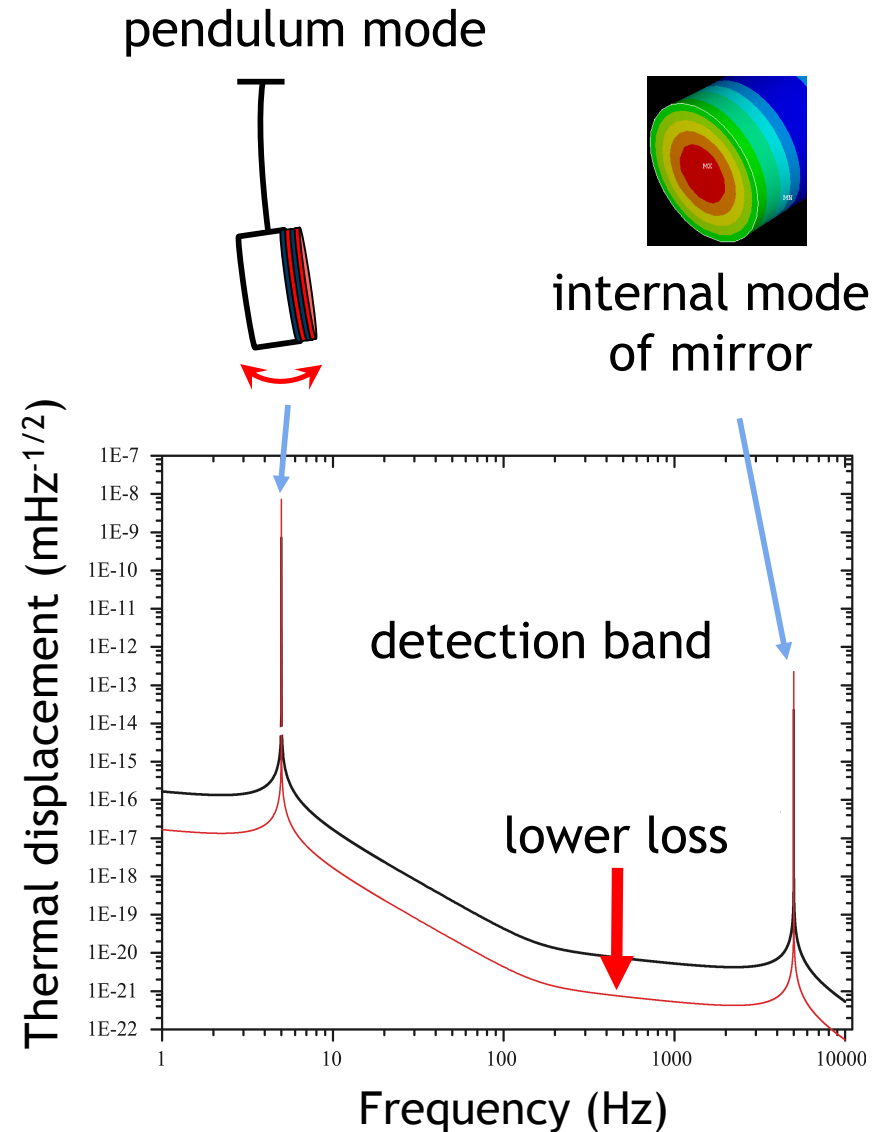
- Surface loss:** dislocations, un-terminated dangling bonds and micro-cracks on the pristine silica surface
- Thermoelastic loss:** heat flow across the fibre due to expansion/contraction leads to dissipation.
- Bulk loss:** strained Si-O-Si bonds have two stable minima which can redistribute under thermal fluctuations.

A.M. Gretarsson et al., Phys. Rev. A, 2000
 G. Cagnoli and P.A. Willems, Phys. Rev. B, 2002
 P.A. Willems, T020003-00
 M.Barton et al., T080091-00-K
 A. Heptonstall et al., Phys. Lett. A, 354, 2006
 A. Heptonstall et al., Class. Quant. Grav, 035013, 2010

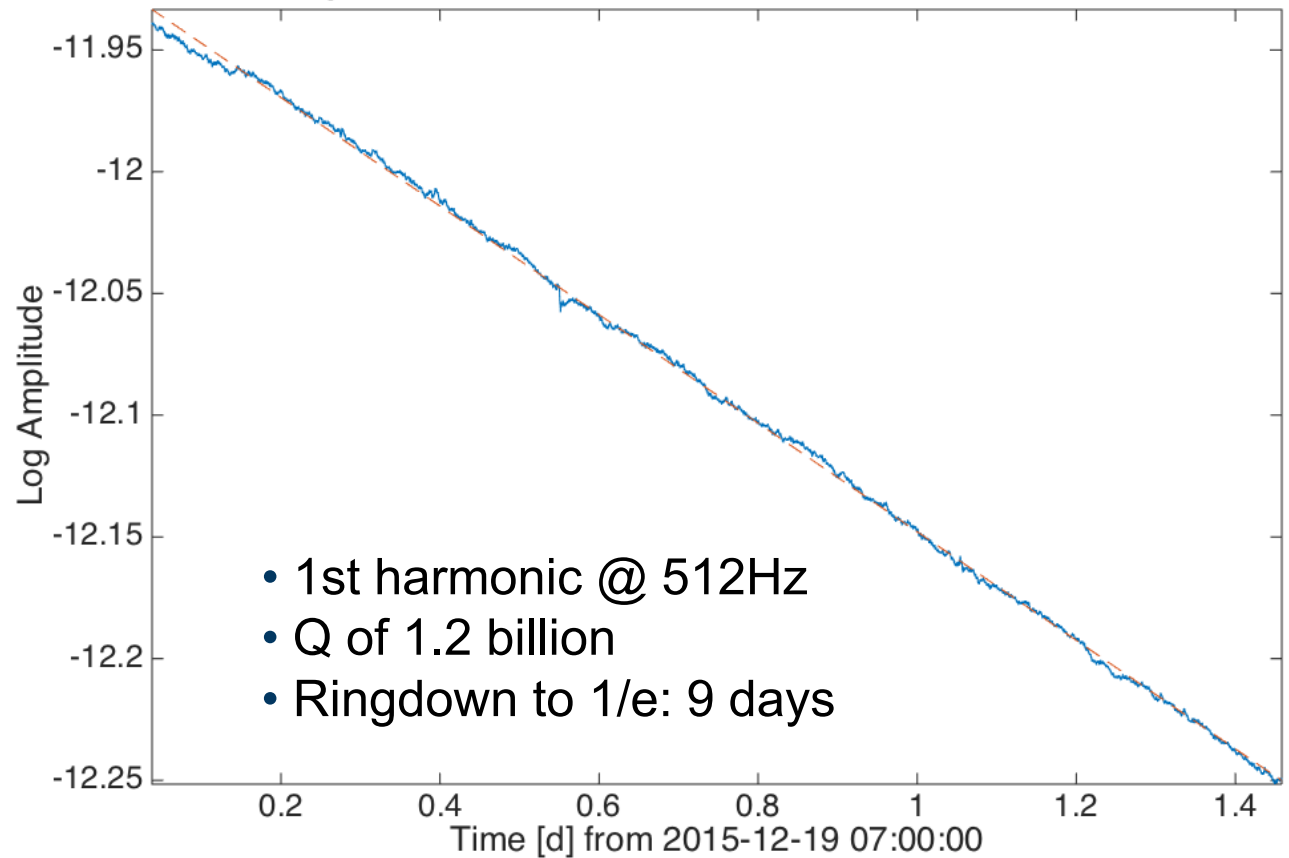
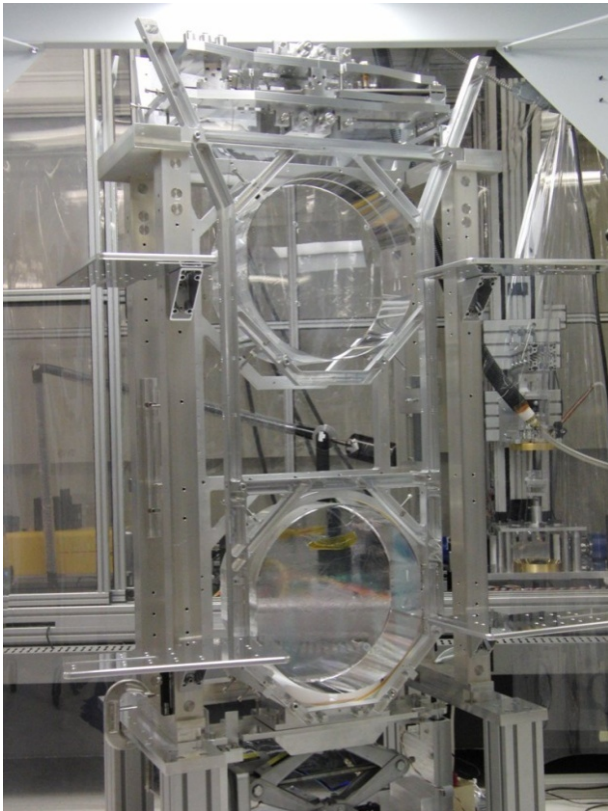
- Thermal energy ($k_B T$) drives resonant modes
- Mechanical loss is energy dissipation by internal friction in material

$$\frac{\Delta\omega}{\omega_0} = \phi = \frac{1}{Q}$$

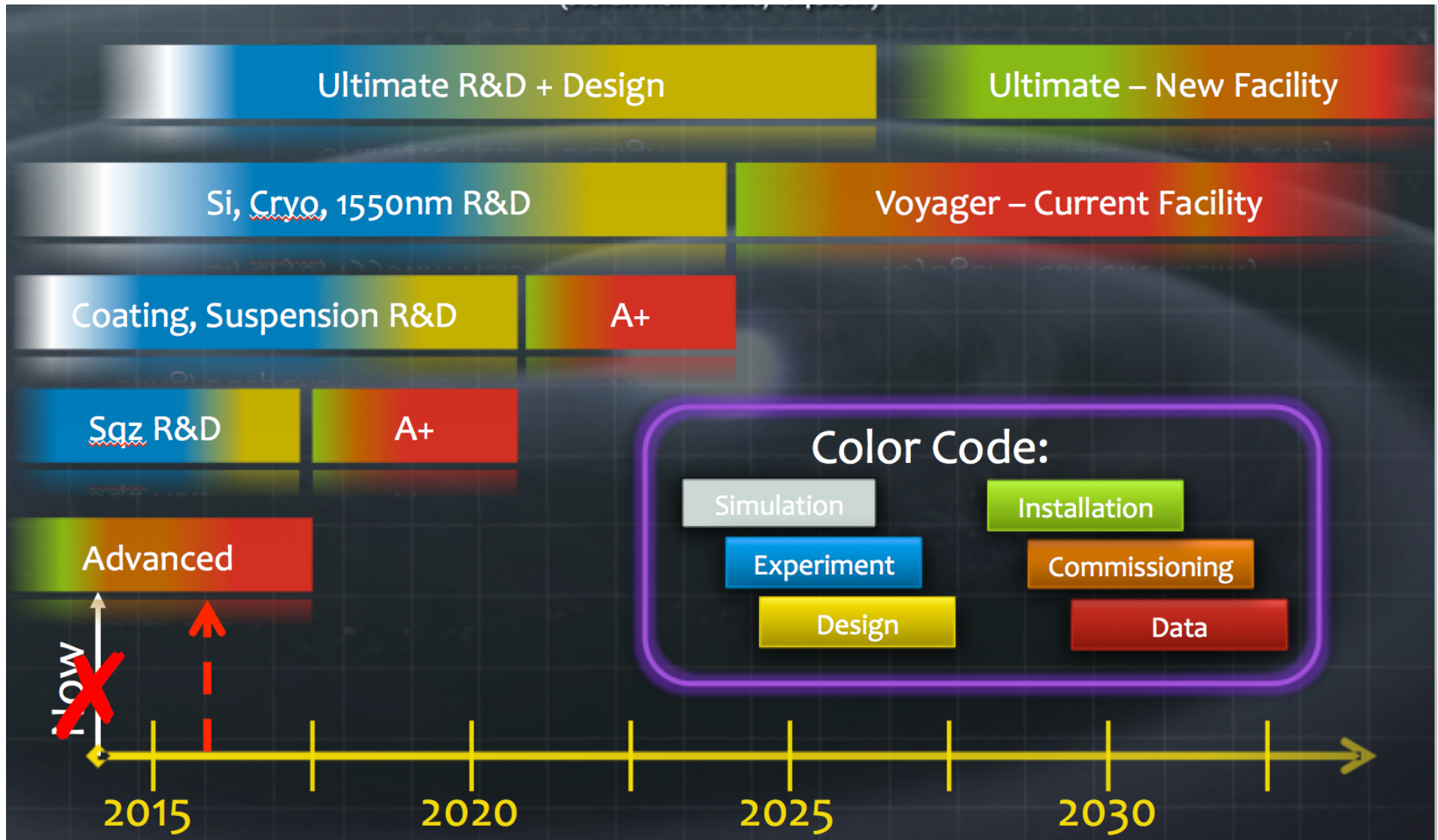
- Lower loss material \rightarrow lower off-resonance thermal noise
- Use of fused silica ($\phi < 10^{-7}$) mirror substrates and suspension fibres in room temperature GWDs



- ANSYS predictions of violin mode quality factors are in good agreement with ringdown measurements => accurate thermal noise model



Current/Future Outlook (aLIGO)



Sensitivity Comparison

